

LOGIC IS NOT LOGIC

Jean-Yves Béziau

Abstract

In this paper we discuss the difference between *logic as reasoning* and *logic as a theory about reasoning*. In the light of this distinction we examine central questions about history, philosophy and the very nature of logic. We study in which sense we can consider Aristotle as the first logician, Descartes's rejection of syllogistic as logical, Boole rather than Frege as the initiator of modern logic. We examine also in this perspective the unfolding of logic into logic and metalogic, the proliferations of logic systems, the questions of relativity and universality of logic and the position and interaction of logic with regards to other sciences such as physics, biology, mathematics and computer science.

1. Anatomy of logic

11. Logic in the shadow

The word "logic" is a common word part of ordinary language, but the adjective "logical" is more frequently used. So the meaning of the word "logic" is generally understood through the adjective "logical". But what is logical and what logic is, the layman doesn't exactly know. As for many words, there is a semantic wavering: the same word can mean different things more or less contradictory; "logic" is no exception, even in the mouth of logicians. There is vagueness, ambiguity and confusion.

We don't want here just to discuss the meaning of the word "logic". We want to clarify what logic is. We want to put the logic room in order. This is a crucial point because if there is confusion at the level of logic, where shall we meet clarity and understanding? Should we accept global confusion, fall into soft relativism or look for another messiah such as cognitive science?

At some point the world was becoming a logical world. People like Wittgenstein, Carnap, Tarski were architects of such a world. Carnap wrote a book with the suggestive title *The logical structure of the world* (1928). Renewing the Aristotelian project these people had the idea that logic is essential, that it is the basis of science and rationality. In

this spirit, the Poles were using the expression “methodology of deductive sciences” as synonymous to “logic”.

About 100 years later this logical world has vanished. We are living neither in a Fregean conceptual paradise, nor in a Russellian type heaven. Who is guilty? Gödel with his incompleteness theorems? Church with his undecidability result? Or Steve Jobs shamelessly exposing his apple in supermarkets all around the world?

Gödel’s and Church’s results can be seen as the failure of Leibniz’s program¹² to build a big system that we can use to think without thinking, in the same way that we can clean our clothes using a washing machine without degrading our hands. But this failure is a happy end to the best-of-the-worlds story. These results are good news: human mind cannot be reduced to an algorithm and in showing that logicians have developed the theory of computation. They gave birth to machines, not replacing human beings, but releasing their brain from an activity that only supporters of Deep Blue may consider as the reflect of intelligence.

Maybe the vanishing of logic is due to the success of computers. Logic is now in the shadow of computer science. Technology is prevailing over science. But we cannot forget the root of computer science: its marvelous fruits are by-products of logic. However, even if logic does not reduce to computer science, this latter has changed the face of logic in a positive lifting. At the dawn of modern logic there was a tendency to try to construct big architectonic logic systems describing everything, solving all the problems. Computer scientists have broken this prehistoric trend being guided by efficiency rather than by megalomania. This has led to many different complementary logical systems. But we have to be careful not to get lost in such a jungle and to keep in mind what logic is. Logic has to do with rationality, it is not only a bouquet of efficient but limited tools.

¹ About Leibniz’s project, see the excellent book by Louis Couturat, *La Logique de Leibniz* (1901).

² Gödel’s comments are the following: “In 1678 Leibniz made a claim of the universal characteristic. In essence it does not exist: any systematic procedure for solving problems of all kinds must be nonmechanical.” (Wang, 1996, 6.3.16); “My incompleteness theorem makes it likely that mind is not mechanical” (Wang, 1996, 6.1.9).

12. Logic medley

To point out the ambiguity surrounding the very nature of logic and the way the word “logic” is used, let us have a look at what some famous thinkers have written about it.

<i>Logic</i> is the anatomy of thought.	John Locke	ca 1700 <i>Un sourced</i>
That <i>logic</i> has advanced in this sure course, even from the earliest times, is apparent from the fact that, since Aristotle, it has been unable to advance a step, and thus to all appearance has reached its completion.	Immanuel Kant	1787 Preface of the Second Edition of <i>Critique of Pure Reason</i>
The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed; to give expression to them in the symbolical language of a calculus, and upon this foundation to establish the science of <i>logic</i> .	George Boole	1853 <i>An Investigation of the laws of thought</i>
Man has such a predilection for systems and abstract deductions that he is ready to distort the truth intentionally, he is ready to deny the evidence of his senses only to justify his <i>logic</i> .	Fyodor Dostoyevsky	1864 <i>Notes from the underground</i>
Contrariwise, if it was so, it might be; and if it were so, it would be; as it isn't, it ain't. That's <i>logic</i> .	Lewis Carroll	1871 <i>Through the looking glass</i>
Bad reasoning as well as good reasoning is possible; and this fact is the foundation of the practical side of <i>logic</i> .	Charles Sanders Peirce	1877 <i>The fixation of belief</i>
<i>Logic</i> takes care of itself and all what we have to do is to look and to see how it does it.	Ludwig Wittgenstein	1914 <i>Journal</i>
Pure <i>logic</i> is the ruin of the spirit	Antoine de Saint-Exupéry	1942 <i>Flight to Arras</i>
With the discovery of the conventional and relative character of <i>logic</i> , human spirit has burned his last idol.	Louis Rougier	1955 <i>Traité de la connaissance</i>
If one wishes to speak about the atomic particles themselves one must either use the mathematical scheme as the only supplement to natural language or one must combine it with a language that makes use of a modified <i>logic</i> or of no well-defined logic at all.	Werner Heisenberg	1958 <i>Physics and philosophy</i>

It is obvious that in these quotations the word “logic”, independently of style and personal views, is used with different meanings. In the following sections we will try to disentangle the logic meanings of this table.

13. Logic and logic

Beyond the paradoxical claim “Logic is not logic”, there is an important distinction: *logic as reasoning* and *logic as the study of reasoning*. This distinction is quite similar to the distinction between History as the series of events and history as the science which studies these events, *History* being the object of study of *history* (see e.g. Woolf, 2011). To keep this parallel in mind we can use the word “Logic” for reasoning and “logic” for the science which studies reasoning, *Logic* being the object of study of *logic*. This is a nice “differance”, pointing the close connection between the two sides of the logical coin.

For many sciences the two sides of the coin are generally clearly linguistically separated, although the distinction ranges from few letters to different words. A radical difference of words is sometimes due to a language shift as in the case of biology and physics. This is also the case of logic: when we say that logic is the science of reasoning, the name of the object of study - “reasoning” - is a word completely different from the name of the science of it - “logic” -, “reasoning” being the Latin word for logic as reasoning.

Here is a table showing differences and variations:

SCIENCE	OF
Biology	Living organisms
Physics	Matter
Linguistics	Languages
Sociology	Society
Poetics	Poetry
Anthropology	Human beings

Why in the case of history people are using the same word, making only and not always a graphical differentiation? And why in logic people are making no difference, graphical or not, favouring the confusion between a science and its object of study. Such a

mess looks similar to what happens with cooking. But in this case the confusion can be justified by a theory/practice rave mix leading to delicious omelettes. Is logic an art like cooking? This is in fact the French conception of it according to the title of the famous book by Arnauld and Nicole: *Logic or the art of thinking* (1662). To go on saming with examples, when someone is studying dance, it is in general not in a contemplative mood, it is for dancing. Dance is an art and the practice of it, the difference here is a substantive/verb difference not necessarily graphically expressed; we can say: “Mary is studying dance to dance”. What about John? We will say: “John is studying logic to reason”. In this formulation the distinction is explicit because there is no “logic” verb, the closer we can get to a redundant formulation is: “John is studying logic to be logical”. But is it really the goal of John when studying logic?

In history, dance and logic, there is on the one hand an activity performed by human beings (we are ignoring here dance history of logical ducks), on the other hand a theory about this activity produced by the same mammals. The connection is strong because there are human beings on both sides of the coin; this is a characteristic of human sciences (but is logic a human science?). A connection that can be understood as an interaction, considering a theory/practice duality.

In history the difference is stronger because rarely the historian will work in order to practice History. But the reason why the difference is not so strong and nearly the same word is used is because the object of historical science is not so objective. There are many different stories, in style and focus. Sometimes we may wonder if they are referring to the same History. In particular History is lost in a “once upon the time”. When did History start? Not right at the beginning, because there is also pre-History with pre-historical men and women, generally not confused with specialists of pre-History, the pre-Historians.³ Historians are also not confused with the first historical human beings, but maybe there is here a secret connection. When and where historical human beings did emerge? There is no clear answer to the question, this reflects the unreality of History. Settling, housing, agriculture, painting, writing, all these activities may be considered as activities surrounding the birth of historical human beings, but maybe they are too much bourgeois,

³ Notions such as *protohistory* (Otte 2008) and *deep history* (Smail 2008) have also been introduced.

emphasizing immobility and comfort rather than the strong stream of History with conquests, revolutions and crises. According to a quite different view, History started in the year 0 or 1, and will never end, a mathematical vision of the world ... A more rational view has been promoted by Heidegger. He would rather say that History started with historical science, with Herodotus and Thucydides, the first historians (see e.g. Shanske 2007). Their objective stories did change History by contrast to mythologies perpetuating karmic circles. Can we similarly claim that Logic started with Aristotle? This is a question we will examine in the next section after comments about the Magrittean character of the title of our paper.

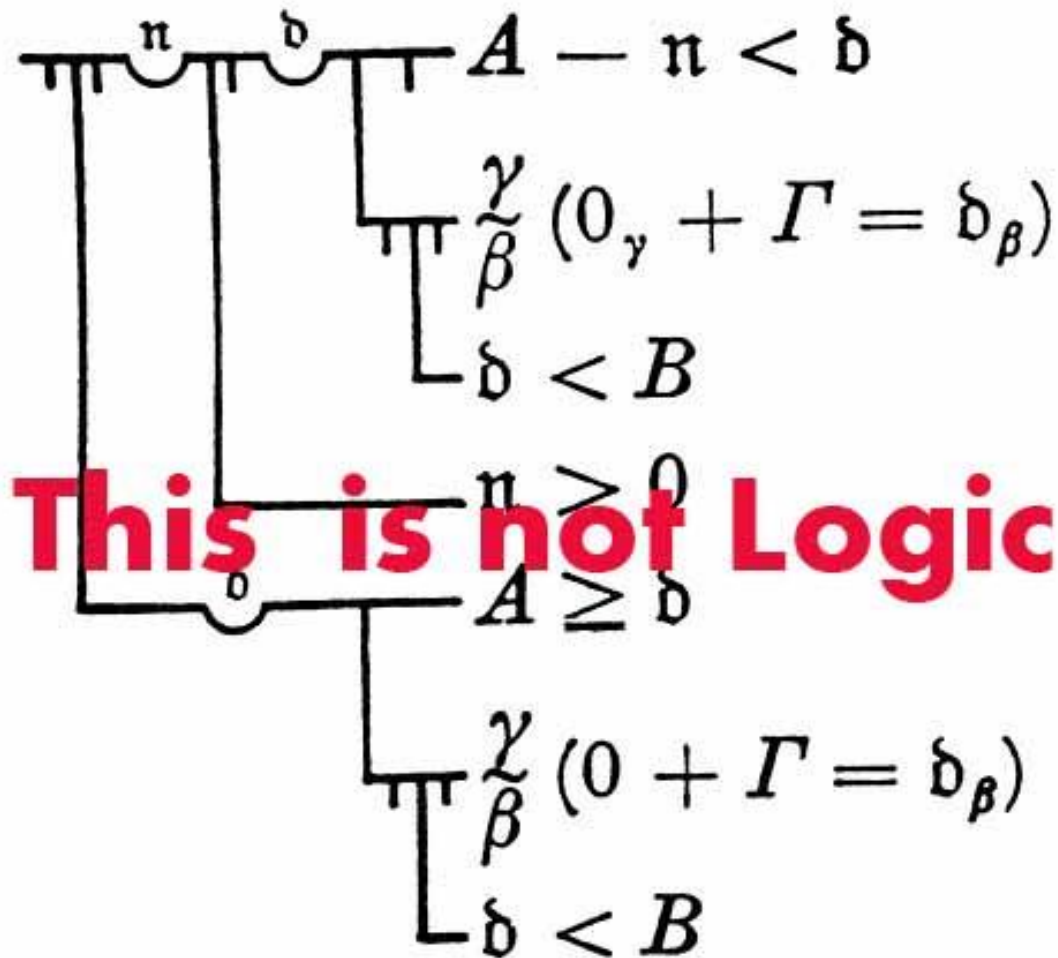
The paradoxical claim “Logic is not logic” remembers Magritte’s paradoxical claim: *Ceci n’est pas une pipe* (*This is not a pipe*). This claim appears in his most famous painting. The name of this painting is not *This is not a pipe* but *La trahison des images*, meaning *The treachery of images*. Magritte explicitly deals with the ambiguity of pictorial representation. Someone may think at first sight that the pipe in the painting of Magritte is more similar to a real pipe than logic science⁴ is similar to reasoning. But pictorial similarity is really a treachery; it is one of the most powerful illusions - visual illusion. The painted pipe is in fact very different from a real pipe, as one can easily understand if he tries to use it for smoking. Of course the word “banana” has not the same taste as a real banana, but the contrast between the thing and its representation is not so strong, since there is no resemblance between the two.

By claiming that “Logic is not logic” we want to stress both the similarity and the difference between logic as reasoning and logic as a science. At first, not paying attention to the scriptural difference or thinking that the capital “L” is due to the beginning of the sentence, this claim may sound like a real contradiction such as “life is not life”, the converse of the declamation “life is life”, a successful song. Such declamation, with many variations, such as “black is black”, is not a tautological claim, the idea is to emphasize the

⁴ We are using here the expression “logic science” as synonymous to “science of logic”, in the same way that “logic theorem” is used as synonymous to “theorem of logic”. “Logic science” therefore means logic as a science. The expression “science of logic” is ambiguous because it can be interpreted as “science of *Logic*” or “science of *logic*”. The latter should be interpreted similarly to “science of biology”, which is synonymous to “biology” tout court, just emphasizing that biology is a science. It is in this way that the expression “science of logic” is used by Boole in the table of our logic medley.

very nature of the thing, which may be indefinable. The famous historian of logic Jean van Heijenoort used to claim: “Life is not first-order, life is not second order: Life is life.” (Anellis, 1994, p.45). But what is the meaning of the antilogical claim “life is not life”? This may stress that life is not what we usually think it is.

We finish this flowering of our discourse with a picture of a Magrittean flavour. The picture below anticipates our discussion in the next sections pointing out that the *Begriffsschrift* is not logic as reasoning – difficult to practice it – but a (pictorial) theory of reasoning.



2. Evolution of logic

21. Logical animals

Many people would say that Aristotle is the first logician. But do they think that he was the first man to reason? No, generally they think he was the first to develop a science of reasoning. And they are right,⁵ but we have to understand the full story. For this it is useful to have in mind the distinction *Logic/logic* and also to merge in classical Greece. The word “logic” derives from a word typical of the Greek culture, the word “logos”, which has no equivalent in other languages. There are four main meanings in its semantic network: *relation, language, reason, science* (Later on, in the Bible, logos became God – cf. John 1:1⁶). The table below describes the situation with examples.

LOGOS 4 MEANINGS	EXAMPLES
relation	irrational numbers (not a <i>relation</i> between natural numbers)
language	neologism (new <i>word</i>)
reason	<i>rational</i> animals λογικό ον
science	anthropology (<i>science</i> of human beings)

Maybe one could claim that modern first-order logic is the full realization of the *logos*: it is the science of reasoning describing relations with a language. But let us come back more than two thousand years ago. Some people argue that mathematics started with the proof of the irrationality of square root of two by the Pythagoreans (see e.g. Dieudonné, 1987). They consider that this was the first mathematical proof. A central feature in this proof is the use of the reduction to the absurd. We can say that with the reduction to the absurd we have a new way of reasoning, a new *Logic*, maybe the birth of *Logic*, tracing here the difference between rational and non-rational human beings. This would be contrary to Aristotle’s definition of human beings as *rational animals* (literal translation:

⁵ Bochenski who has extensively studied Indian logic (see Bochenski 1956) says that Indian logicians can be credited to inventions as valuable as Aristotle’s ones, the difference being that it took them several centuries (see Bochenski 1990).

⁶ The standard translation of John 1:1 is: “In the beginning was the Word, and the Word was with God, and the Word was God”, where “Word” is the translation of “logos”: Ἐν ἀρχῇ ἦν ὁ λόγος, καὶ ὁ λόγος ἦν πρὸς τὸν θεόν, καὶ θεὸς ἦν ὁ λόγος. Gödel speaks of a “rational principle behind the world” (Wang, 1996, 8.4.10).

logical animals). This definition implies that rationality is an essential feature of human beings and that consequently they have always been rational.

According to Szabó (1969, 1994), the reduction to the absurd was first used by the philosophers of the Eleatic school, Parmenides and Zeno. Szabó's thesis based on a detail historiographic study is a confirmation of an idea defended more than one century before by Schopenhauer, emphasizing that rationalism was a philosophical attitude based on the rejection of sense data. But Schopenhauer thinks that rationalism, in particular mathematical rationalism supported by the reduction to the absurd, is wrong. Eighty percent of the proofs in Euclid's *Elements* are based on the reduction to the absurd and Schopenhauer is not afraid to say that this method is properly absurd. In his first book *On the fourfold root of the principle of sufficient reason* (1813), he presents some new proofs of theorems of geometry, based on pictures, not on "absurd reasonings". Schopenhauer's approach has influenced Wittgenstein and the Intuitionists. Schopenhauer's philosophy of mathematics is not something new. For him the way to escape the empirism/rationalism dichotomy is the Kantian theory of pure intuitions of space and time: we can reason directly and safely about space using some intuitive pictorial proofs, this is much better than kilometers of reduction to the absurd proofs. Schopenhauer has been inspired by Kant, but he has developed much more the theory, elaborating the distinction and relation between logic and mathematics, being probably the first to introduce the terminology "metalogical". He uses this word to qualify the fourth class of truths, corresponding to the fourth root of the principle of sufficient reason (see Béziau 1993).

Let us come back to good old Greece. Greeks were rational animals. But why did these rational animals introduced *irrational* numbers? This is one of the mysteries of the *logos*. Pythagoreans had the belief that everything can be explained with natural numbers or *relations* between such numbers, *rational* numbers. But their belief was dismissed by the *logos* through a *reasoning* based on the reduction to the absurd, the proof of the *irrationality* of square root of two. So they were rationally led to irrationality or better: the reduction to the absurd became the key of the *logos*, opening the kingdom of rationality to

irrational numbers and much more. Summarizing: Logic was born in the Pythagorean boat and its true face is the reduction to the absurd.⁷

Now what is the relation between Aristotle and such *Logic*? Aristotle didn't practice it and Aristotelian *logic* is not a science of it. Moreover mathematicians never used syllogistic to practice mathematics. It is interesting to compare the situation with tragedy: Aristotle elaborated a theory of tragedy in his *Poetics* based on the great tragedies of his time. He didn't write tragedies but his theory has been used - in Hollywood it is still a basis for screenwriting (see e.g. Tierno, 2002).

Aristotle was nevertheless a great promoter of logic, as a tool and as a science. He had the idea that *proof* is the central characteristic of science. He was interested to develop logic as a general methodology of science but also to avoid sophisms. Modern logic is in continuity with the Aristotelian perspective: logic appears as methodology of science and as critical thinking. What has been rejected is Aristotelian logic as a given theory describing reasoning: syllogistic. The main reason of this rejection is that it is not giving an accurate description of mathematical reasoning.

First-order logic is a better description, but one may argue that first-order logic, like syllogistic, is more a theory of reasoning than an effective way of reasoning. Nevertheless in modern times there had been a better interaction between *Logic* and *logic*. Despite the rejection of the new logic science by some mathematicians, logic has changed mathematics: looking closer we see that modern mathematics is directly connected to modern logic. If we consider that mathematics is (part of) reasoning, we can say that *logic* has changed *Logic*. And vice versa *Logic* has changed *logic*, because logic as mathematical reasoning has been applied to develop logic science – that was not the case of Aristotle's syllogistic, not based on mathematics. Algebra, topology, category theory have been applied to develop logic, as the science of reasoning.

The highest development of the first stage of modern logic, the mathematical foundations wave, is model theory. Model theory is a beautiful interaction between *Logic* and *logic* establishing a vital link between mathematical structures and the way we reason

⁷ About the relation between irrational numbers and irrationality, see the very interesting book by G.-G. Granger, *L'irrationnel* (1998).

about them. The second stage of modern logic connected with the proliferation of non-classical logics is also an interaction between *Logic* and *logic*: people are constructing systems of logic and applying them, we have here a theory/practice duality like in dance and other arts and techniques - this is *techno-logic*.

More than ever we are logical animals.

22. Logical cuisine

An important philosopher who was against the Aristotelian trend was Descartes. We can see Descartes as the father of modern philosophy, in particular breaking the Aristotelian tradition. Descartes is not anti-rationalist, but he is against the rationalism of Aristotle and neo-Aristotelian philosophy (scholastic). Descartes is promoting a new kind of rationalism. The distinction between logic as reasoning and logic as science is useful to understand this shift of rationalism. For many people, Cartesian means logical. So if we say that Descartes didn't like logic, people may be surprised. This Cartesian paradox is clear up if we explain that Descartes didn't like logic as a science, in particular syllogistic. He thought that to be logical it is not necessary to use syllogistic, it can even disturb our reasoning in the same way that if we are trying to apply a theory to walk, we will not walk in a better way, but maybe fall.

So what is the Cartesian way? Descartes believes in a natural disposition: "Good sense is, of all things among men, the most equally distributed". To reason we don't need a theory of reasoning, *Logic* doesn't need *logic*. Descartes wrote two books with suggestive titles: *Rules for the direction of mind* (1628) and *A Discourse of a method - For a method for the well guiding of reason, and the discovery of truth in the sciences* (1637), but these books don't develop a science of reasoning.

Descartes emphasizes that we must think clearly and distinctly and summarizes his methodology in four principles he presents in contrast with logic as syllogistic: "Instead of the great number of precepts of which logic is composed, I believed that the four following would prove perfectly sufficient for me, provided I took the firm and unwavering resolution never in a single instance to fail in observing them" (Descartes, 1637). Here is a table presenting Descartes's four precepts (the nicknames on the left column are ours):

DESCARTES 4 PRECEPTS	
<i>Clarity</i>	Never to accept anything for true which I did not clearly know to be such; that is to say, carefully to avoid precipitancy and prejudice, and to comprise nothing more in my judgment than what was presented to my mind so clearly and distinctly as to exclude all ground of doubt.
<i>Division</i>	To divide each of the difficulties under examination into as many parts as possible, and as might be necessary for its adequate solution.
<i>Ascension</i>	To conduct my thoughts in such order that, by commencing with objects the simplest and easiest to know, I might ascend by little and little, and, as it were, step by step, to the knowledge of the more complex; assigning in thought a certain order even to those objects which in their own nature do not stand in a relation of antecedence and sequence.
<i>Exhaustivity</i>	To make enumerations so complete, and reviews so general, that I might be assured that nothing was omitted.

We can consider these precepts as promoting logic, considered as good rational thinking. The aim of Descartes is not to theorize, developing a science of reasoning, but to practice: *The Discourse of a method* includes applications of the method to Dioptrics, Meteors and Geometry. Descartes also applies his methodology to philosophical issues: *proving* his own existence and also the existence of God. Such proofs are not chains of syllogisms, Descartes clearly states that *cogito ergo sum* is not the conclusion of a syllogism. Descartes is promoting Logic as rational thinking, free of Barbarian syllogistic.⁸

Blaise Pascal has on this respect a position quite similar to Descartes. For Pascal the highest way of reasoning is the one we find in geometry, based on an obvious natural methodology and we can say bye bye to Barbara, Celarent and all their syllogistic friends, which are of no use to develop right thinking and avoid sophisms. Pascal wrote in *The Art of Persuasion* (1656): “To discover all the sophistries and equivocations of captious reasonings, they have invented barbarous names that astonish those who hear them ... It is not Barbara and Baralipon that constitute reasoning. The mind must not be forced; artificial and constrained manners fill it with foolish presumption, through unnatural elevation and vain and ridiculous inflation, instead of solid and vigorous nutriment. And one of the principal reasons that diverts those who are entering upon this knowledge so much from the true path which they should follow, is the fancy that they take at the outset

⁸ A good presentation of Descartes’s views on logic is (Gaukroger 1989).

that good things are inaccessible, giving them the name of great, lofty, elevated, sublime. This destroys everything. I would call them low, common, familiar: these names suit them better; I hate such inflated expressions.”

By opposition to the sophistry of syllogistic, Pascal defends 8 rules, “the true ones”, that are “simple, artless, and natural”. We present them in the following table:

PASCAL 8 RULES	
<i>Rules for Definitions</i>	Not to undertake to define any of the things so well known of themselves that clearer terms cannot be had to explain them.
	Not to leave any terms that are at all obscure or ambiguous without definition.
	Not to employ in the definition of terms any words but such as are perfectly known or already explained.
<i>Rules for Axioms</i>	Not to omit any necessary principle without asking whether it is admitted, however clear and evident it may be.
	Not to demand, in axioms, any but things that are perfectly evident of themselves.
<i>Rules for Proofs</i>	Not to undertake to demonstrate any thing that is so evident of itself that nothing can be given that is clearer to prove it.
	To prove all propositions at all obscure, and to employ in their proof only very evident maxims or propositions already admitted or demonstrated.
	To always mentally substitute definitions in the place of things defined, in order not to be misled by the ambiguity of terms which have been restricted by definitions.

Can we say that Descartes/Pascal position is typically French? Is this the quintessence of a French vision of logic extending up to Poincaré (1905-06) not afraid to qualify the new logic as *pipi de chat*? Maybe clarity of thought is, with champagne and Roquefort, a typical French specialty - a good mix indeed.

But this French logic cuisine does not necessarily reduce to a regional delicacy. Tarski presented a conference at the 9th *International Congress of Philosophy* organized in Paris in 1937 entitled “Sur la méthode deductive” and published in French in the *Annals* of this event under the same title. This paper is the same as Chapter 6 of Tarski’s bestseller *Introduction to Logic and to the Methodology of the Deductive Sciences* (1936) entitled “On the Deductive Method”.

Alfred Tarski was a great admirer of Blaise Pascal. He considers that modern logic, as methodology of deductive science, is very similar to Pascal's methodology of *The Art of persuasion*. The first section of the chapter "On the Deductive Method" is entitled "Fundamental constituents of a deductive theory—primitive and defined terms, axioms" and the following footnote is attached to the first sentence of this section (p.109): "Ideas which are closely related to those presented in this section can be found in earlier literature. See, for instance, the opusculum (posthumously published), *De l'esprit géométrique et de l'art de persuader*, of the great French philosopher and mathematician B. PASCAL (1623-1662)."

Like Pascal, Tarski thinks that the model of reasoning has to be found in Euclid's geometry. Tarski sees modern logic as a renewal of this method. For Tarski there is continuity between Euclid, Pascal and Hilbert. The deductive method as the trinity *Definition-Axiom-Proof*, promoted by Pascal and considered as the central architecture of the deductive method by Tarski, is not due to Aristotle. It was developed by mathematicians and it is rather a methodology than a science of reasoning. But Tarski, like Aristotle,⁹ considers that reasoning does not reduce to mathematical reasoning, modern logic "arose originally from the somewhat limited task of stabilizing the foundations of mathematics. In its present phase, however, it has much wider aims. For it aspires to relate to the whole of human knowledge. In particular, one of its goals is to perfect and to sharpen the deductive method, which not only has a central place in mathematics, but in addition, in just about every domain of intellectual endeavor, serves as an indispensable tool for deriving conclusions from accepted assumptions." (Tarski, 2004, p.IX) However Tarski, differently to Aristotle, considers that this methodology is fundamentally based on mathematics: "Logic (the deductive method) applies to every science and in particular to itself, which should for this reason be regarded as a mathematical discipline" (p.112).

We will in the next section study more this *reflexive character of logic*.

⁹ Tarski like Aristotle had a strong interest for biology - this was his first love, and he encouraged his friend J.H.Woodger to develop the methodology of biology, see (Woodger, 1937)

23. Logical buildings

Hilbert has coined the word “metamathematics”. This monstrous name is not so much in use nowadays, people prefer to use the expression “proof theory” that Hilbert was using synonymously. Hilbert’s follower, Nicolas Bourbaki started his famous multivolume treatise *Eléments de Mathématique* by claiming “qui dit mathématique, dit démonstration” (see Bourbaki 1970). For Hilbert the substance of mathematics are proofs. So for him the study of what mathematics is, is the study of what a proof is. To perform such study we have to go above mathematics, hence the bigname “metamathematics”, where the prefix “meta” is understood as “above”.

During some years the word “metamathematics” had an extensive use, in fact it was used as a synonymous to “logic”. It was the time of Hilbert’s reign. In 1952 Kleene wrote a book called *Introduction to metamathematics* which became a fundamental textbook of logic. Eleven years later was published a book with a punny name: *The mathematics of metamathematics* (1963). Is this punny title the beginning of the end of metamathematics? Surely there is a shift of perspective. This title is provocative because the idea of Hilbert’s program was to develop metamathematics using a different methology as *within* mathematics, in particular in view of proving consistency results. The standard view is that proofs are made of strings of symbols and that we stay in the denumerable to manipulate them. But Gentzen, member of the Hilbert’s school, was himself changing things, at the time of Gödel’s second incompleteness result he proved the relative consistency of arithmetic, using transfinite induction.

In Poland there was a different perspective right at the start. The Polish school was much influenced by the work of Schröder on algebra of logic (see Woleński, 1992). In Poland grew a tradition of using mathematics to develop logic as any branch of mathematics. As pointed out by Tarski, there is a certain reflexive character in logic, but this is not necessarily seen as a vicious circle, it can be viewed as an elevating spiral. Logic is not the foundation on which the building of mathematics or science is erected; it is rather its architecture. A different perspective that may be understood through the definition by Tarski of logical notions are those invariant under any transformation (Tarski, 1986).

“Metamathematics” is related with Hilbert; in Poland a connected word has been much popular, the word “metallogic”. What is metallogic? If logic is synonymous to metamathematics, metallogic is a nickname for metametamathematics. So we are facing here a three story building, that can be represented by the following table, placing our difference between *Logic* and *logic*:

	Name	What it is
3 rd Floor	Metallogic	theory of the theory of reasoning
2 nd Floor	logic	theory of reasoning
1 st Floor	Logic	reasoning
THE 3-STORY BUILDING OF LOGIC		

Generally people consider rather a two story building. This is not without ambiguity. Are they collapsing the 2nd and the 3rd floors? In a proof system, say LK, a theorem is something proven within the system LK, it is an object of the 2nd floor. A metatheorem such as the cut-elimination theorem is an object of the 3rd floor, it is a result about LK, not a result proved in LK. One may want to study the system where cut-elimination is proved. This can lead to a 4th floor, but logicians will rather try to go down than up, trying to reduce the 3rd floor to the second floor. Gödel’s work is typically on this direction, with the arithmetization of syntax and the notion of proof turning into a modal operator within a logical system.

But the reduction to a two story building can be based on a different view. Sometimes people use the word “metallogic” to speak of a theory whose object is logic, not clearly making the distinction between reasoning (*Logic*) and a system describing this reasoning (*logic*). In this case they are collapsing the 1st and 2nd floors.

The prefix “meta” has become popular in philosophy, maybe due to a complex of superiority. Philosophers are talking about “metaethics”, “metaphilosophy” and even “metametaphysics”. On the other hand nowadays logicians are not using so much metawords. Maybe things are clear enough without climbing above, paradise in on earth: if John says to Mary he is studying logic, she will generally understand that he is studying some logical systems describing reasoning. Maybe she would have been more impressed if

he had said he was studying metalogic. Then she would have invited him to the hell of a metadance.

3. Cosmology of logic

31. Logic, logic and logics

The XXth century has been very fruitful for logic, in quality and in quantity. Uncountable logical systems were born during this baby boom period. And this fertility boost is going on. Is it a multiplicity of *logics* or of *Logics*? To properly answer this question we must examine the relation between a (system of) logic, and the Logic it describes.

To have examples at hands and to better understand the problem, let us first try to classify the multiplicity of logical systems. Our objective here is not to present an exhaustive classification of all existing and possible logics but to show that there are different ways of slicing the logic cake, that we will illustrate with typical specimens.

We can start with the distinctions between *deviation* and *expansion* of classical propositional logic, deviation means that the properties of standard classical connectives are modified, expansion means there are some additional connectives.¹⁰

DEVIATION/EXPANSION	
Deviations	Intuitionistic logic Relevant logic
Expansions	Modal logic Causal logic

Let us note that here “modal logic” is not the name of a logical system but the name of a class of logical systems: there are thousands of systems of modal logic. Moreover the dichotomy deviation/expansion is not necessarily exclusive: we can have a relevant modal logic. A variation that does not appear explicitly in this table is a rather vertical variation, that can be understood through the distinction propositional logic / first-order logic and that is more fully expressed by the following table based on the example of classical logic but that can be applied to intuitionistic logic and other logics.

¹⁰ Haack (1974) uses the terminology “extensions”, which is quite ambiguous, we use here “expansions” by reference to the use of this word in model theory.

GRADES	
Subsystems	Positive classical propositional logic Full classical propositional logic
Supersystems	Many-sorted classical first-order logic Second order classical logic

There is not only one way to generate a system of logic. And we can consider that in some sense two techniques correspond to two different systems. We can classify logic systems according to the way they have been generated. *Substructural logics* are defined in this perspective: they are logics constructed by modifying the structural rules of sequents systems. Here is a simple table describing classification by techniques:

TECHNIQUES	
Proof	Hilbert systems Sequents systems
Semantics	Logical matrices Kripke structures

It is important to note that these different ways of slicing the cake don't lead to the same results. For example if we call Kripke logics, logic systems generated by the technique of Kripke structures, this class of logics is not the same as the class of modal logics, because on the one hand there are some modal systems that cannot be characterized by a Kripke structure, and on the other hand Kripke structures can be used to develop logics without modalities, such as a logic of implication.

Now let us examine if the multiplicity of *logics* (logic systems) corresponds to a multiplicity of *Logics* (ways of reasoning).

One may argue that there is only one Logic and that the multiplicity of logics is not against this oneness: multiple logic systems can be viewed as descriptions of the many aspects of this big Logic. In physics, there are many theories, this does not necessarily mean that there are different physical realities. These theories can be viewed as describing different aspects of the same reality. In logic, when we have expansions, such as a modal logic, or grade variations such as a many-sorted logic, we can argue that these logics are all

describing the same Logic. And also in the case of deviations, as pointed out by da Costa (1980) in the case of paraconsistent logics: a paraconsistent negation can be seen as an additional operator.

In physics we may have different concurrent theories, for example Ptolemy's theory is not the same as Newton's theory. They are about the same reality but one seems closer to reality than the other one. We can say the same of first-order logic comparing it to syllogistic. Here again this does not mean that we have different physical realities or different Logics, just different ways to look at the same thing.

But the case in logic is more difficult than the case of physics, because logic is both a normative and a descriptive theory.¹¹ The normative/descriptive distinction is useful to understand different positions non-classical logicians may have. When someone says that classical logic is not "real reasoning", this may be understood in two different ways. On the one hand one may argue that classical logic is not the right description of reasoning as it is, on the other hand one may argue that classical logic is not the right way of reasoning. In the first case one is speaking of classical *logic*, in the second case of classical *Logic*.

A typical example of the second case is Brouwer. He thought that classical *Logic* was wrong: this is not the way we should reason in mathematics, for him the right way is intuitionistic *Logic*. Brouwer, like other mathematicians, had no interest in logic as a science of reasoning, he was not interested to develop intuitionistic *logic*. This was done by his student Heyting and people generally think that Heyting's *logic* is a good description of Brouwer's *Logic* (see van Stigt, 1990 and Moschovakis, 2009).

32. The relativity of logic

The answer to this question depends on the two sides of the logic coin and the relation between them, but here we shall also talk about a hidden third dimension: the logic of reality.

¹¹ Lewis Carroll defines logic as the "science of reasoning rightly", see (Moktefi, 2008). This classical definition shows the normative character of logic. But this formulation is quite ambiguous: its meaning may range from "the art of reasoning (rightly)" to "a theory of what correct reasoning is".

Without being a post-modernist for whom the theory of relativity has the same value as Tupi-guarana's cosmology, one may think that science is relative because it is always changing, an idea which seems quite natural nowadays. The main idea is that science is progressing. Theories are slowly improving, or sometimes there are some breaks: a theory is rejected as false and replaced by a totally different one. Syllogistic was rejected, but some people see first-order logic as an improvement of syllogistic.

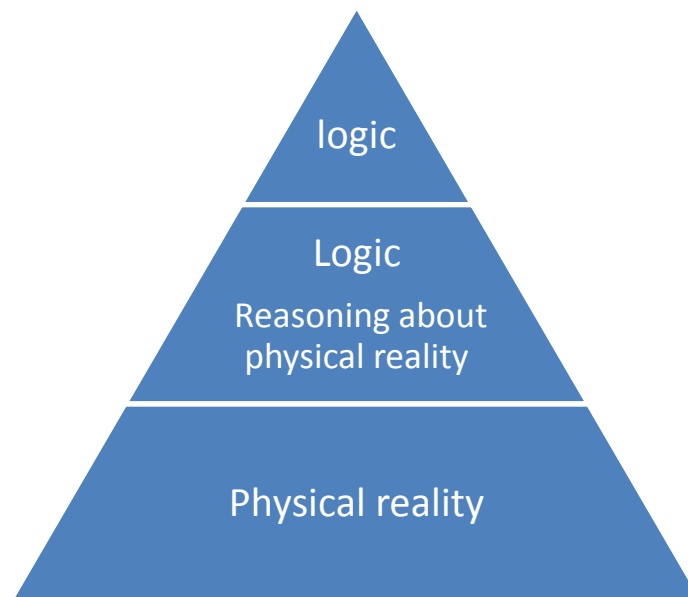
It is true that there are still nowadays people revering the religious scientificity of the enlightenment period, a time when a scientific theory could be seen as absolutely true. This was in particular the case of Newton's theory, a truth absoluteness in harmony with the intrinsic absoluteness of Newtonian physics: absoluteness of space, time and laws of nature. Some people may similarly think that first-order logic is an absolutely true theory perfectly describing perfect reasoning. Newtonian absoluteness was seriously challenged by the changes of modern physics: both at the microscopic level, with quantum physics, and at the macroscopic level, with relativity theory. And some physicists are arguing that not only time and space are relative but also that the laws of nature are changing (see e.g. Barrow 2002).

We have to face the very nature of objective reality. The physicists from the Copenhagen school have not necessarily rejected the idea of an objective reality; in fact Bohr's complementary theory is a way to save such reality. But due to the results of quantum physics there is the idea that reality cannot be known as it is, independently of some human experimentations which modify it. Since physics is a major science defining the house we are living in, the universe we are merged in, all these astonishing changes of physics have an impact in our way to consider science and reality in general.

And in fact since the start there was an interaction between modern physics and modern logic. Heisenberg has argued that our reasoning based on classical logic cannot describe the phenomena of the microscopic world (see e.g. Heisenberg 1958). We may try to change our logic and new logics have been proposed.¹² The situation can be interpreted in different

¹² The universe of quantum logics is in continuous expansion. An interesting logic system dealing directly with Heisenberg's uncertainty principle was proposed by Paulette Février in 1937 in Paris at the same congress where Tarski was talking about the deductive method (see Février, 1937). Tarski was a good friend of Paulette Février during many years (see Feferman and Feferman, 2004). In this book the authors also

ways: one may think that the logic of physical reality is not classical logic. If we consider that logic is reasoning, how can we speak of “the logic of physical reality”? Can we say that a stone is reasoning? For Aristotle the principle of contradiction was a law of reality, it was the structure of the world. Nowadays we have a less anthropomorphic view of reality, we don't see negation as part of reality. Reality is not black or white. It is not tricolor either. Negation, classical or neo-classical, is a tool to conceive reality. We can speak of the logic of physical reality in this sense, this logic of reality is a way of reasoning about reality, and we can develop a system of logic describing it, indirectly describing physical reality.¹³ Here is a picture summarizing the stratification:



Physics is not the only science with theories radically changing our vision of science and the world. Changes started in fact before, in biology, with the theory of evolution, rejecting the idea of living beings of a permanent type, human beings included. If our brain is evolving, and if we think that the brain is an organ strongly connected with reasoning, it is

explain how Tarski encouraged Patrick Suppes to organize in 1957 at Berkeley a big event on the axiomatic method with special reference to geometry and physics (Paulette Février was there). After this successful event, Tarski decided to launch the series of congresses LMPS (Logic, Methodology and Philosophy of Science) still going on.

¹³ Compare with the views of Joe Brenner in his book *Logic in reality* (2008).

natural to think that reasoning changes, that *Logic* is relative. And also *logic* as a science: cognitive scientists may rightly think we have to replace the study of the Aristotelian organon by the study of another organ: the brain; not only human brain but also the brain of other animals, to see the resemblance and the difference, to eventually find what characterizes the logicity of human brain. Recent discoveries tend to show that the logicity of human beings is not so different from the logicity of other animals – it was proved many years ago that dogs can perform disjunctive syllogisms, without reading Aristotle (see Aberdein, 2008).

We are nowadays in a situation completely different from the time when people like Kant had the idea that Logic was a set of fixed laws of thought perfectly described by Aristotelian logic. Boole also had the idea that Logic was made of laws of thought but he didn't think that syllogistic was a good description of these laws. He started to use mathematics to describe them, and then everything started to change by an interaction between the object of study and the theory, an interaction between *Logic* and *logic*.

For this reason we can consider that Boole is the true generator (rather than creator) of modern logic by opposition to Frege. Frege with his *Begriffsschrift* produced a static picture of reasoning, it is a beautiful cliché but it cannot be used. Human beings are not Boolean, human brains are not necessarily working according to the laws of a Boolean algebra. But a science of reasoning based on mathematics has led us to conceive new systems of logic, for example with a negation obeying neither the principle of contradiction, nor the law of excluded middle. A logic that can be implemented in computers (see e.g. Belnap, 1997) but also that can be used by human beings to reason in a different way about reality.

The first section of Chapter 6 of *Philosophy of logic* by Quine (1979) is entitled, “change of logic, change of subject”. Quine, against deviant logics, claims: “If sheer logic is not conclusive, what is? What higher tribunal could abrogate the logic of truth functions or of quantification?”. We may wonder if Quine is talking here about *Logic* or about *logic*. Quine says that people dealing with non-classical negations are making a linguistic confusion; they don't know what they are talking about. Now can we say that

people by claiming that the earth was spherical rather than plane were not denying the doctrine, but just changing the subject?

The logic of truth functions is a system of logic dated from the beginning of the XXth century. It has some quality and it has some defects. To think that it describes the true *Logic* because it is a nice system would be like arguing that everything in the universe is spherical because spherical astronomy is a beautiful theory. Luckily enough our reasoning is not a slave of some laws of thought described by binary truth functions. Using our reasoning we can develop some new logical systems changing *Logic*.

33. The universality of logic

Let us first examine the question of universality of logic, as a science. It is not necessarily contradictory to argue that science is both relative and universal.¹⁴

We can say that universality is a fundamental and characteristic feature of science, in the sense that: (1) science is not a private business, it is objective, not subjective, not a question of taste; (2) science explains not the idiosyncrasies of a particular phenomenon, but some general patterns of phenomena. We can see that logic, since Aristotle, has these two universal features. The first feature is explicitly manifested through a theory like syllogistic which is a system with a set of rules. The second feature is also clear since syllogistic is concerned with all kinds of reasonings. On the other hand Aristotle thought it was impossible to develop a science of history, because for him there was no universality beyond the particulars turbulences of human societies – easier to develop logic, biology and meteorology.

Science is concerned with a double *ALL*, *ALL* minds and *ALL* objects. It is interesting to make a connection with the universal quantifier in logic. Chuaqui and Suppes (1995) have shown that classical physics can be described with a first-order logic theory with only universal quantifiers.

But a given science is not purely universal. A science like biology does not apply to all phenomena, it applies to a *portion* of reality, or better an *aspect* of reality. Biology is the science of life, so tautologically it does not apply to non-living things like a stone, but also

¹⁴ Rougier was defending the relativity of logic and at the same time the unity of science (see Marion 2011).

it does not necessarily apply to all aspects of living beings, for example the basic concepts of biology are generally not used to explain the mathematical activity of a living being. So biology is not completely universal from outside. Neither it is from inside. There are different branches of biology, using different methods and having different objects of study. Studying whales is not the same as studying mushrooms, different also from studying trees. There are things in common and differences. Someone who is studying the origin of life is concerned with all living beings, but not directly with all aspects of all living beings. Within biology there are different levels of universality.

In the logic science, similar things happen: one may be concerned with some particular kinds of reasoning. There are many different kinds of reasoning, each having its specificity. When study legal reasoning, it is interesting to develop a deontic system of logic with an operator of obligation. This operator may have some common properties with other logical operators like necessity, but someone studying mathematical reasoning doesn't need to work with a system of logic with an obligation operator. To develop a big "universal" system of logic encompassing all the varieties of reasoning would be quite monstrous. But, as a biologist concerned with the very nature of living beings, one may be interested in the very nature of reasoning beyond all particular kinds of reasoning. In this case one may study some general concepts, like the notion of consequence relation. This goes in the direction of "universal logic" (see Béziau 2006) understood similarly to universal algebra (see Birkhoff 1987).

Such universal logic is not a universal system of logic, in the same way that general linguistics is not a universal language. General linguistics is the study of the common features of all languages. It is universal in this sense. Human beings are using thousands of different languages. We can ask the question "is language universal?" meaning "is there something in common beyond all particular human languages?" We can have a positive answer to this question, but this does not mean that what is beyond all particular human languages is itself a language. General linguistics may be considered as universal because having as object of study something which is universal, an object which is not a universal language, but the universal features of all languages. Because of the universal character of

its object of study and because of its methodology, general linguistics has the universal characteristic of a science, but without being itself a universal language.

Similarly it is possible to develop logic as a science whose object of study are the general features of all kinds of reasonings, not a universal reasoning. This logic science is universal because its object of study is universal, but it is neither a universal logical system nor a universal way of reasoning (for more details see Béziau 2010).

Jean-Yves Béziau

Universidade Federal do Rio de Janeiro

jyb@ifcs.ufrj.br

4. Bibliography¹⁵

- Aberdein, A. (2008) ‘Logic for dogs’, IN S.D.Hales (ed), *What philosophy can tell you about your dog*, Chicago: Open Court, pp. 167-181.
- Anellis, I.H. (1994) *Logic and its history in the work and writings of J. van Heijenoort*, Ames: Modern Logic Publishing.
- Arnauld, A. and Nicole, P. (1662) *La Logique ou l’art de penser (Logic or the art of thinking)*, Paris: Savreux.
- Barrow, J.D. (2002) *The Constants of nature: From alpha to omega - The Numbers that encode the deepest secrets of the universe*, New York: Pantheon Books.
- Belnap, N. (1977) ‘A useful four-valued logic’, IN M.Dunn (ed), *Modern uses of multiple-valued logic*, Boston: Reidel, pp.8-37.
- Béziau, J.-Y. (1993) ‘La critique Schopenhauerienne de l’usage de la logique en mathématiques’, *O Que Nos Faz Pensar*, 7, pp.81-88.
- . (2006) ‘13 questions about universal logic’, *Bulletin of the Section of Logic*, 35, pp.133-150.
- . (ed) (2010) Special Issue of *Logica Universalis* on the topic “Is logic universal?”, number 2, volume 4.
- . (ed) (2011) *An Anthology of universal logic*, Basel: Birkhäuser.
- Birkhoff, G. (1987) ‘Universal algebra’, IN Rota, G.-C. and Oliveira, J. S. (eds.), *Selected Papers on Algebra and Topology by Garret Birkhoff*, Birkhäuser, Basel, pp. 111-115.
- Bochenski, J.M. (1956) *Formale Logik*, Karl Alber, Fribourg (English translation *A History of formal logic*, Indiana: Notre Dame, 1961).
- . (1990) *Entre la logique et la foi*, Editions noir sur blanc, Montricher.
- Boole, G.: (1854) *An Investigation of the laws of thought, on which are founded the mathematical theories of logic and probabilities*, London, and Cambridge: Macmillan & Co.
- Bourbaki, N. (1970) *Théorie des ensembles*, Paris: Hermann.

¹⁵ Thank you to Arthur Ronald de Vallauris Buchsbaum for his support in finding references with the help of electronic devices and also to Amirouche Moktefi.

- Brenner, J.E. (2009) *Logic in reality*, Berlin: Springer.
- Carnap, R. (1928) *Der Logische Aufbau der Welt*, Felix Meiner, Leipzig (English translation *The Logical Structure of the World - Pseudoproblems in Philosophy*, University of California Press, 1967).
- Carroll, L. (1871) *Through the Looking-Glass, and What Alice Found There*, London: Macmillan.
- da Costa, N.C.A. (1980) *Ensaio sobre os fundamentos da lógica*, São Paulo: Hucitec.
- Chuaqui, R. and Suppes, P. (1995) 'Free-variable axiomatic foundations of infinitesimal analysis: A fragment with finitary consistency proof', *The Journal of Symbolic Logic*, 60, 122–159.
- Couturat, L. (1901) *La Logique de Leibniz – D'après des documents inédits*, Paris: Félix Alcan.
- Descartes, R. (1637) *Discours de la méthode (Discourse on the method)*, Leyde.
- . (1628) *Regulae ad directionem ingenii (Rules for the direction of mind)*, published in Amsterdam, 1701.
- Dieudonné, J. (1987) *Pour l'honneur de l'esprit humain*, Paris: Hachette. (English translation: *Mathematics – The music of reason*, Berlin: Springer, 1992).
- Dostoyevski, F. (1864) *Zapiski iz podpolya (Записки из подполья)* (English translation: *Notes from Underground*).
- Feferman, S. and Feferman, A.B. (2004) *Tarski: Life and Logic*, Cambridge: Cambridge University Press.
- Février, P. (1937) 'Les relations d'incertitude d'Heisenberg et la logique', IN *Travaux du IXème Congrès International de Philosophie*, VI, Paris : Hermann, pp. 88-94.
- Frege, G. (1879) *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens (Concept Notation, the Formal Language of the Pure Thought like that of Arithmetics)*, Louis Nebert, Halle a. S.
- Gaukroger, S. (1989) *Cartesian logic – An essay of Descartes's conception of inference*, Oxford: Oxford University Press.
- Granger, G.-G. (1998) *L'Irrationnel*, Paris: Odile Jacob.
- Haack, S. (1974) *Philosophy of logics*, Cambridge: Cambridge University Press.

- Heidegger, M. (1927) *Sein und Zeit (Being and Time)*, Tübingen: Niemeyer.
- Heisenberg, W. (1958) *Physics and philosophy - The revolution in modern science*, New York: Harper.
- Hilbert, D. (1965) *Gesammelte Abhandlungen*, 3 volumes, New York: Chelsea.
- Kant, I. (1787) *Kritik der reinen Vernunft (Critique of pure reason*, second edition).
- Kleene, S.C. (1952) *Introduction to metamathematics*, Amsterdam: North-Holland.
- Kneale, W. and Kneale, M. (1962) *The Development of logic*, Oxford: Oxford University Press.
- Marion, M. (2011) ‘Louis Rougier on the relativity of logic – an early defence of logical pluralism’, IN (Béziau 2011).
- Moktefi, A. (2008) ‘That’s logic!’, *Daresbury Lewis Carroll Society Journal*, 8, pp.23-22.
- Moschovakis, J.R. (2009) ‘The logic of Brouwer and Heyting’, IN D.M.Gabbay and J.Woods (eds), *Handbook of the History of Logic – Volume 5*, Amsterdam: Elsevier.
- Otte, M. (2008) *La Protohistoire*, Bruxelles: De-Boeck.
- Pascal, B. (1657) *De l’esprit géométrique et de l’art de persuader (The art of persuasion)*.
- Peirce, C.S. (1877) ‘The fixation of belief’, *Popular Science Monthly*, pp.1-5.
- Poincaré, H. (1905-06) ‘Les mathématiques et la logique’, *Revue de Métaphysique et de Morale*, 1905, p. 815-835, 1906, p. 17-38, et p. 294-317.
- Quine, W.V.O. (1970) *Philosophy of logic*, New Jersey: Prentice Hall.
- Rasiowa, H. and Sikorski, R. (1963) *The Mathematics of Metamathematics*, Warsaw: Polish Academy of Sciences.
- Rougier, L. (1955) *Traité de la connaissance*, Paris: Gauthiers-Villars.
- . (1941) ‘The Relativity of logic’, *Philosophy and Phenomenological Research*, 2, pp.137-158, reprinted in (Béziau 2011).
- de Saint-Exupéry, A. (1942) *Pilote de guerre (Flight to Arras)*, New York: Editions de la maison française.
- Schopenhauer, A. (1813) *Über die vierfache Wurzel des Satzes vom zureichenden Grunde*, Rudolstadt (*On the fourfold root of the principle of sufficient reason*).
- Schanske, D. (2007) *Thucydides and the philosophical origin of history*, Cambridge: Cambridge University Press.

- Smail, D.L. (2008) *On Deep history and the brain*, Berkeley: University of California Press.
- van Stigt, W.P. (1990) *Brouwer's Intuitionism*, Amsterdam: North Holland.
- Szabó, A. (1969) *Anfänge der griechischen Mathematik*, Budapest: Akademiai Kiado (English translation: *The Beginnings of Greek mathematics*, Dordrecht: Kluwer, 1978).
- . (1994) *Die Entfaltung der griechischen Mathematik*, Mannheim: B.I. Wissenschaftsverlag.
- Tarski, A. (1936) *O logice matematycznej i metodzie dedukcyjnej*, Lwów and Warsaw: Ksiaznica-Atlas (English translation; we refer here to the 4th edition by Jan Tarski: *Introduction to Logic and to the Methodology of the Deductive Sciences*, Oxford: Oxford University Press, 1994).
- . (1937) 'Sur la méthode déductive', in *Travaux du IXe Congrès International de Philosophie*, VI, Paris: Hermann, pp.95-103.
- . (1986) 'What are logical notions?', *History and Philosophy of Logic*, 7, pp.143-154.
- Tierno, M. (2002) *Aristotle's Poetics for screenwriters: Storytelling secrets from the greatest mind in Western civilization*, New York: Hyperion.
- Wang, H. (2006) *A Logical journey - From Gödel to philosophy*, Cambridge: MIT Press.
- Wittgenstein, L. (1961) *Notebooks 1914-1916*, New York: Harper.
- Woleński, J. (1989) *Logic and Philosophy in the Lvov-Warsaw School*, Dordrecht: Kluwer.
- Woodger, J.H. (1937) *The Axiomatic method in biology, with appendices by Alfred Tarski and W.J.Floyd*, Cambridge: Cambridge University Press.
- Woolf, D. (2011) *A Global history of history*, Cambridge: Cambridge University Press.

5. Acknowledgments

I don't remember when exactly I started to discuss the distinction between logic as reasoning and logic as the science of reasoning using the calligraphic difference "Logic" and "logic", but I can objectively trace it back via slides as far as a talk I presented at the *Santa Fe Institute* in New Mexico, September 29, 2005. Thank you to David Krakauer my host at the Santa Fe Institute.

I have good souvenirs of this trip to New Mexico followed by a trip from San Francisco to Los Angeles via Highway Number 1. My thanks go also to the late Herb Enderton who invited me to give a talk at UCLA on the same topic on October, 7, 2005, and who was also my host during a Fulbright stay at UCLA ten years before, in 1995.