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The Theory of the Formal Discipline and the Possible Interpretations of Conditionals: Material Versus Defective Conditionals[1]

Miguel López Astorga

Universidad de Talca
Instituto de Estudios Humanísticos
milopez@utalca.cl

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Abstract
Attridge and Inglis try to check whether or not the 'Theory of Formal Discipline' is correct. This theory states that learning mathematics improves logical reasoning, and Attridge and Inglis review it by means of an experiment. Their conclusion is that, indeed, learning mathematics improves conditional inferences causing that conditionals are interpreted as defective. In this paper, I analyze Attridge and Inglis's experiment and hold that its results can be interpreted in a different way and that hence does not really prove that learning mathematics lead to defective interpretations of conditionals. Equally, the paper includes a brief reflection on how the mental models theory can explain the results achieved by Attridge and Inglis.

1 Introduction

Attridge and Inglis (2013) focus on the discussion, open by Plato, about whether mathematics improves reasoning. Thus they carry out an experiment and interpret that their results certainly demonstrate that idea, which is known as the 'Theory of the Formal Discipline'. However, they hold that, in addition, their results lead to other interesting findings. The improvement made by mathematics consists of understanding conditionals as defective, and not as material.

Nevertheless, Attridge and Inglis's (2013) results do not necessarily invalidate other alternative hypotheses and other interpretations of them (based on a material interpretation) are also possible. In my opinion, the problem is that Attridge and Inglis do not consider other interpretative possibilities and it is what causes that they consider that studying mathematics leads one to interpret conditionals as defective. This does not mean that the task used by them is not appropriate. In fact, it is a task that gave Evans, Clibbens, and Rood (1995) interesting results. The real problem is how Attridge and Inglis (2013) interpret the results that they obtain by means of that task. Their results certainly show that learning mathematics to some extent improves conditional reasoning (the trend to biconditionality decreases and fallacies tend to

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disappear) but there are also crucial aspects involved in conditional reasoning that seem to be worse. In particular, those aspects refer to the use of the rule of *Modus Tollens* (from now on, MT), which appears not to be appropriately applied by the participants studying mathematics in Attridge and Inglis's experiment.

Maybe the problem is that Attridge and Inglis's (2013) participants are teenagers that have only had one year of advanced mathematics (post-compulsory level in England). In particular, they compare the results obtained by those participants in conditional reasoning tasks before and after completing such studies. In fact, they also compare these students' results to the results achieved by students that have not studied advanced mathematics. However, for this paper, as it can be noted below, only mathematics students' results are relevant.

I say that the participants can be the problem because it is obvious that their characteristics limit the scope of Attridge and Inglis's (2013) results. Such results are not useful for determining whether or not learning mathematics improves logical reasoning, but only for showing whether or not the mathematics post-compulsory level in England improves logical reasoning. Therefore, their conclusions and the conclusions that I will expose in this paper must be considered under this limitation. In this way, it should not be forgotten that those conclusions do not refer to learning mathematics in general, but to a concrete type of learning mathematics.

On the other hand, Attridge and Inglis (2013) also state that their results cannot explained by the mental model theory (Byrne & Johnson-Laird, 2009; Johnson-Laird, 1983, 2001, 2006, 2012; Johnson-Laird & Byrne, 2002; Johnson-Laird, Byrne, & Girotto, 2009; Orenes & Johnson-Laird, 2012). >From a semantic perspective, this theory (from now on, MMT) claims that human reasoning works by means of explicit and implicit models, but, according to Attridge and Inglis (2013), their findings are hard to explain based on it. Nevertheless, in my view, what happens is that Attridge and Inglis do not take certain essential aspects of MMT into account.

In this way, next pages will show the interpretation problem that I see in Attridge and Inglis's (2013) research, explain why their results do not demonstrate that learning mathematics is linked to a defective interpretation of conditionals, and, finally, expose the reasons why it can also be said that the semantic framework of MMT is coherent with such results. However, first of all, it seems opportune to describe in more details Attridge and Inglis's (2013) experiment.

2 Logical rules, fallacies, and interpretations of conditionals

Attridge and Inglis (2013) think that conditional can be interpreted in four ways. It can be taken as material, biconditional, defective, and conjunction. I briefly explain what these interpretations are (from now on, I assume that '1' stands for truth and '0' stands for falsehood):

- Material: This is the classical interpretation from logic and, as it is well-known, it establishes that $v(p \rightarrow q) = 0$ only if $v(p) = 1$ and $v(q) = 0$. Otherwise $v(p \rightarrow q) = 1$.
- Biconditional: The causes that make conditionals are considered as biconditionals have been studied in great details in the literature, and the conditional perfection phenomenon (e. g. van der Auwera, 1997a, 1997b; Geis & Zwicky, 1971; Horn, 2000; Moldovan, 2009) is especially interesting in this way. Nonetheless, what is important here is that $v(p \leftrightarrow q) = 0$ when either $v(p) = 0$ and $v(q) = 1$ or $v(p) = 1$ and $v(q) = 0$. Otherwise $v(p \leftrightarrow q) = 1$.
- Defective: This interpretation is related to probabilistic logic (e. g., Adams, 1998; Adams & Levine, 1975), but, in this paper, the basic aspect of it that needs to be considered is that, under a defective interpretation, $v(p \rightarrow q) = 1$ if $v(p) = 1$ and $v(q)$

$= 1$, and $v(p \rightarrow q) = 0$ if $v(p) = 1$ and $v(q) = 0$. If p is not present (i. e., $\neg p$ happens), $p \rightarrow q$ is not a relevant relation.

- Conjunction: As it can be easily understood, under this interpretation, $p \rightarrow q$ is considered as $p \wedge q$, and, as it is also well-known, $v(p \wedge q) = 1$ when $v(p) = 1$ and $v(q) = 1$. Otherwise $v(p \wedge q) = 0$.

The case is that Attridge and Inglis (2013) provide equivalences between these four interpretations and two logical rules and two fallacies related to conditional. The two rules are the rule of *Modus Ponens* (from now on, MP) and MT, and the two fallacies are affirming the consequent (from now on, AC) and denying the antecedent (from now on, DA). Their equivalences are the following:

- The material interpretation only allows using MP and MT.
- The biconditional interpretation allows using MP, MT, AC, and DA.
- The defective interpretation only allows using MP.
- The conjunctive interpretation only allows using MP and AC.

Thus, taking this equivalences into account, Attridge and Inglis (2013) carried out an experiment in which, among other tasks and exercises, their participants had to solve reasoning tasks related to MP, MT, AC, and DA. In those tasks abstract conditional propositions (which referred to numbers and letters) were used as first premise. The second premise was the antecedent of the rule (MP), the denial of its consequent (MT), its consequent (AC), or the denial of its antecedent (DA). In this way, obviously, the conclusion was the consequent of the rule (MP), the denial of its antecedent (MT), its antecedent (AC), or the denial of its consequent (DA), and participants' task was to indicate, responding 'yes' or 'no', whether or not, in each of those four cases, the conclusion follows from the two premises.

As mentioned, Attridge and Inglis's (2013) participants solved those tasks twice, before and after completing a year of advanced mathematics, and the results relevant for this paper obtained by them were these: before completing the level, the participants tended to accept in large numbers the conclusions corresponding to the four inferences (a proportion between 0.7 and 0.8 in a scale from 0.2 to 0.8). Nevertheless, after completing it, they only tended to admit MP (a proportion between 0.5 and 0.6 in a scale from 0.2 to 0.8). This fact was interpreted by Attridge and Inglis (2013) as clear evidence that learning mathematics improves logical reasoning. However, in their view, the most important finding was that the improvement was related to the defective interpretation (which was that linked only to MP), and not to the material interpretation.

Nonetheless, as said above, I think that Attridge and Inglis's (2013) results do not show a global or general improvement of their participants' logical abilities. Their interpretation of their results is problematic, since it is also possible to interpret such results from an approach based on the material interpretation and to state that mathematics post-compulsory level in England only improves certain aspect of logical reasoning (it limits the tend to interpret conditional as biconditional). In my opinion, a material interpretation of such results can lead one to say that other aspects (those related to MT) are not improved by that mathematics level, and that, in a sense, they even worsen. This is explained in the next part.

3 The material interpretation and the rejection of MT

The main problem in the analysis of their results offered by Attridge and Inglis (2013) is that the link between the defective interpretation and the use of only MP is not obvious. When, in a conditional reasoning task, an individual responds 'yes' (that is, that the conclusion follows from the premises), his (or her) answer means that he (or she) thinks that the denial of the conclusion is not possible. The difficulty appears when he (or she) responds 'no' (that is, that the conclusion does not follow from the premises). This situation is a difficulty because we have no information on his (or her) thoughts, and hence we do not know the causes of his (or her) response. Certainly, the defective interpretation can be correct and, when the scenario refers to $\neg p$, he (or she) can respond 'no' because he (or she) considers that scenario to be irrelevant. However, we cannot be sure of that. It is also possible that the participant responds 'no' because he (or she) thinks that the conclusion is false or that the denial of the conclusion is possible. Thus, the acceptance of only MP is not necessarily linked to the defective interpretation. Therefore, it seems legitimate to assume other perspective and to interpret the answer 'no' in a different way in order to give an alternative explanation of Attridge and Inglis's (2013) results.

In my view, beyond the defective interpretation, the best alternative explanation to assume is that the participant responds 'no' because he (or she) thinks that the denial of the conclusion is possible. The answer 'no' does not imply that the individual thinks that the conclusion is necessarily false. Simply, it means that a scenario in which the denial of the conclusion is true is possible. Of course, the participant can respond 'no' because, in his (or her) opinion, the conclusion is false, but, given that we only know that he (or she) chose 'no', all we can state is that he (or she) admits the possibility that the denial of the conclusion is true, since we cannot know for sure whether or not he (or she) considers the conclusion to be false.

>From this perspective, in which the defective interpretation is not considered, Attridge and Inglis's (2013) results can have other meaning. This different meaning can be clear if we think about the logical structure of the tasks used by Attridge and Inglis (2013) and the semantic possibilities linked to both the answer 'yes' and the answer 'no'.

As far as MP is concerned, its premises are $p \rightarrow q$ and p, and the participant must indicate whether or not q follows. If he (or she) responds 'yes', he (or she) is saying that $v(p \land \neg q) = 0$. However, if he (or she) responds 'no', he (or she) is not necessarily saying that $v\ (p \land q) = 0$. We can only be sure that he (or she) thinks that a scenario in which $v(p \land \neg q) = 1$ is possible.

On the other hand, the premises of MT are $p \rightarrow q$ and $\neg q$, and, in this case, the participant must decide whether or not $\neg p$ follows. If he (or she) responds 'yes', he (or she) is stating that $v(p \land \neg q) = 0$. However, if he (or she) responds 'no', he (or she) is not necessarily stating that $v(\neg p \land \neg q) = 0$. We can only be sure that he (or she) thinks that a scenario in which $v(p \land \neg q) = 1$ is possible.

The case of DA is similar. The premises are now $p \rightarrow q$ and q, and participants' task is to decide whether or not p follows. If he (or she) responds 'yes', he (or she) is claiming that $v\ (\neg p \land q) = 0$. However, if he (or she) responds 'no', he (or she) is not necessarily claiming that $v(p \land q) = 0$. We can only be sure that he (or she) thinks that a scenario in which $v(\neg p \land q) = 1$ is possible.

Finally, in DA the premises are $p \rightarrow q$ and $\neg p$, and the participants must answer whether or not $\neg q$ follows. If he (or she) responds 'yes', he (or she) is saying that $v(\neg p \land q) = 0$. However, if he (or she) responds 'no', he (or she) is not necessarily saying that he (or she) thinks that $v(\neg p \land \neg q) = 0$. We can only be sure that he (or she) thinks that a scenario in which $v(\neg p \land q) = 1$ is possible.

Based on these arguments, it is correct to link biconditional to MP, MT, AC, and DA. The answer 'yes' means the following:

- In the cases of MP and MT: $v(p \wedge \neg q) = 0$.
- In the cases of AC and DA: $v(\neg p \wedge q) = 0$.

Therefore, given that $v(p \leftrightarrow q) = 1$ when $v(p \wedge q) = 1$ or $v(\neg p \wedge \neg q) = 1$, and that, by accepting MP, MT, AC, and DA, these last possibilities are not rejected, it can be said that, certainly, an individual that tends to consider those four inferences as valid is an individual that tends to interpret conditional as biconditional.

Equally, it is also appropriate to relate the material interpretation to only MP and MT. The answer 'yes' means the following:

- In the cases of MP and MT: $v(p \wedge \neg q) = 0$

But the answer 'no' means the following:

- In the cases of AC and DA: It is possible that $v(\neg p \wedge q) = 1$.

Therefore, given that $v(p \rightarrow q) = 0$ only if $v(p \wedge \neg q) = 0$, and that this possibility is precisely the only possibility that is explicitly rejected (that is in the cases of MP and MT), it can be said that an individual that tends to consider only MP and MT as valid is an individual that tends to the material interpretation of conditional.

Likewise, it is also right to link the conjunctive interpretation to only MP and AC. The answer 'yes' means:

- In the case of MP: $v(p \wedge \neg q) = 0$.
- In the case of AC: $v(\neg p \wedge q) = 0$.

And the answer 'no' means:

- In the case of MT: In principle, it is possible that $v(p \wedge \neg q) = 1$. However, given that this possibility is forbidden by MP, the answer 'no' can only mean here that $v(\neg p \wedge \neg q) = 0$.
- In the case of DA: In principle, it is possible that $v(\neg p \wedge q) = 1$. However, given that this possibility is forbidden by AC, the answer 'no' can only mean here that $v(\neg p \wedge \neg q) = 0$ as well.

Therefore, given that $v(p \wedge q) = 1$ only if $v(p) = 1$ and $v(q) = 1$, and that all other possibilities are rejected, it can be said that an individual that tends to accept only MP and AC is an individual that tends to interpret conditional as conjunction.

The problem is, as indicated, the relation of the defective interpretation with only MP. Of course, it can be thought that an individual that only accepts MP is an individual that rejects MT, AC, and DA because they are irrelevant (i. e., they refer to scenarios in which p is not). Nevertheless, given that, as commented, we do not know participants' thoughts, it can also be argued that the acceptance of only MP can also be linked to the material interpretation. I explain this idea below.

As said, in the case of MP, the answer 'yes' means that the individual thinks that $v(p \wedge \neg q) = 0$. However, in the other cases, the answer 'no' allows certain possibilities. As also mentioned, in the cases of AC and DA, $v(\neg p \wedge q) = 1$ is allowed, and this fact, evidently, is not a problem. The difficulty is given by MT. In its case, the answer 'no' means, apparently, that the participant accepts as possible a scenario in which $v(p \wedge \neg q) = 1$. Obviously, this is an inconvenience, since the combination $p \wedge \neg q$ is forbidden by MP. Of course, this inconsistency can lead one to think that the material interpretation cannot be linked to the participants that only accepted MP. The material interpretation not only requires the acceptance of MP and the rejection of AC and DA, but also the acceptance of MT. Therefore, this is the point that needs to be explained.

MT is very different from MP. MP is a simple and basic rule, but MT is complex and it cannot be used without other rule: *Reductio ad Absurdum*. If one wants to apply MT, he (or she) needs to adopt p as assumption to apply MP (considering the premise $p \rightarrow q$ and the assumption p), which leads to obtain q, and to note that the premise $\neg q$ is inconsistent with q, and that hence p is not possible. This process is far more complex than that of MP (which only needs one step) and this additional difficulty has been reported in the literature (e. g., Byrne & Johnson-Laird, 2009; López Astorga, 2013). In fact, Attridge and Inglis (2013) also refer to this issue. They state that this is the explanation of the difficulty of MT that the authors that support the idea of a mental logic often raise. They even seem to acknowledge that this explanation can be valid, even though the defective interpretation is assumed. Nonetheless, what is important is that, if this explanation is considered valid, Attridge and Inglis's (2013) results do not prove that mathematical study (at least, the kind of mathematical study corresponding to the post-compulsory level in England) causes a trend towards the defective interpretation. It is possible to interpret conditionals materially and, however, not to use MT. And this is because this last rule is hard to apply.

Thus, from this perspective, it can be said that mathematics only improves logical reasoning in a certain sense. Before the post-compulsory level, students tend to interpret conditionals as biconditionals and, according to Attridge and Inglis's (2013) results, learning mathematics corrects this problem. Nevertheless, logical reasoning ability tends to be worse in other sense after that same level, since students have difficulties to use MT. Such difficulties are not observed before the post-compulsory level because, under the biconditional interpretation, $v(\neg p \wedge \neg q) = 1$, which, apparently, must be noted to apply MT. Maybe it would be interesting to review the syllabi corresponding to the mathematics post-compulsory level. It is possible that they do not include the resolution of problems with processes similar to the application of MT, and that this fact is the cause that mathematics students do not often use this rule after the post-compulsory level.

4 MMT and learning mathematics

The main aim of this paper is not to propose arguments in favor of MMT. My more important goal is only show that Attridge and Inglis's (2013) results do not necessarily mean that mathematical study lead to a defective interpretation of conditionals. However, given that Attridge and Inglis (2013) state that their results are difficult to understand from the basic theses of MMT and that I think that that idea is not correct, it can be opportune to explain briefly, before concluding, why, in my view, Attridge and Inglis's (2013) research does not cause difficulties to MMT.

MMT is a wide theory on human reasoning. Nevertheless, what is relevant for this paper is only its explanation about conditionals. According to MMT, when an individual reasons about conditional propositions, he (or she) considers the possibilities, or models, corresponding to each conditional. Thus, a proposition such as $p \rightarrow q$ has three models:

A.– $p \;\&\; q$

B.– $\neg p \;\&\; q$

C.– $\neg p \;\&\; \neg q$

The key point is that only A is an immediate and explicit model. Both B and C need certain cognitive effort and the action of memory for becoming explicit models. In this way, faced to a conditional, individuals firstly take only one model (A) into account. B and C, as indicated, are models more difficult to consider.

Attridge and Inglis (2013) think that these theses are not coherent with their results. If their participants interpreted conditionals as biconditionals before mathematical advanced level, it means that they considered two models in that moment. In particular, the considered models were A (explicit) and C (implicit). A allowed the application of MP and AC, and C allowed the application of MT and DA. The problem is that, as I understand Attridge and Inglis's arguments, it is not easy for MMT to explain what happens at the later point in time, i. e., the acceptance of only MP. Certainly, it could be argued that C is eliminated after learning mathematics, but, if A is the only model that is considered, MP should not be the only rule that is accepted. AC should be accepted as well. This is, at least in my view, what Attridge and Inglis (2013) seem to mean as regards MMT.

However, I think that arguments such as these are not correct. A relevant datum that must be considered is that, after studying mathematics, a greater tendency to accept MP and AC (i. e., to the conjunctive interpretation) was also observed, which means that mathematical advanced study is somehow linked to the rejection of the biconditional interpretation and the acceptance of the conjunctive interpretation, that is, to the rejection of the model C and the acceptance of only A (the explicit model). Nevertheless, because what was most striking was that the mathematics post-compulsory level leaded the participants to admit only MP, it is obvious that MMP needs to clarify this last fact. In this way, I can say that, in my opinion, it is true that A and C are the models that must be taken into account if the interpretation is biconditional, but the acceptance of only MP can be explained, from the perspective of MMT, as a change of models. Indeed, if the considered models are not A and C, but A and B, MP must be accepted (by virtue of A), MT must be rejected (C is not considered), AC must also be rejected (by virtue of B), and DA must be rejected as well (by virtue of B too). Thus, based on MMT, it can be stated that learning mathematics also improves logical reasoning in a sense, since it causes that conditional relations are not interpreted as biconditional relations (the model B is added). Nonetheless, it is worse in another sense, because the model C is lost and hence it is not possible to accept MT.

Of course, this explanation proposed by me can be questioned by referring to certain theses of MMT. For example, Johnson-Laird and Byrne (2002) state that individuals only have two options: they can consider the explicit model (A) or all the models (A, B, and C). Thus, it does not seem possible that the participants only consider A and B (without C). However, as far as this difficulty is concerned, I can say that my arguments are only an interpretation of MMT that respects their main and basic theses and that, at the same time, is consistent with Attridge and Inglis's (2013) results. After all, MMT allows some models to be blocked in certain circumstances and, according to it, individuals only represent what they believe to be true. In this way, it can be thought that my explanation develops ideas that, although they are not clearly held by the adherents of MMT, do not contradict the core of this theory.

Thus, if my interpretation of MMT is right, it can be said that it continues to be clear that mathematics post-compulsory level in England corrects the mistake of interpreting conditionals as biconditionals. Equally, it can be stated that, from the perspective of MMT, the problem is also MT, since it seems that proofs related to the opposite of what must be proved (which are linked to C) are not practiced by students in post-compulsory level.

5 Conclusions

It is hence obvious that, if we do not assume the defective interpretation, it can be said that mathematical study, or mathematics post-compulsory level in England, only improves logical reasoning in a sense: $p \rightarrow q$ is not interpreted as $p \leftrightarrow q$. The ability of use MT is not improved.

This fact does not mean that the theory of the formal discipline can only be considered true if the defective interpretation is accepted. Maybe the problem is that the kind of mathematics taught in the post-compulsory level is not the type required for improving the use of some logical rules (especially MT). In addition, other possibility is that the post-compulsory level is not enough and that more mathematics levels are necessary for a better use of such rules.

In any case, there is no doubt that Attridge and Inglis's (2013) results do not conclusively prove that learning mathematics lead one to a defective interpretation of conditionals. As shown above, such results are coherent with the idea that their participants continue to interpret conditionals materially after the post-compulsory level. Furthermore, as also commented, MMT is also compatible with their results and can explain them.

Obviously, I cannot deny that the defective interpretation continues to be a possibility that must be taken into account. Undoubtedly, although Attridge and Inglis's (2013) results are consistent with a material interpretation and MMT, it is evident that they are also consistent with the defective interpretation proposed by Attridge and Inglis (2013). In this way, it can only be said that, if mathematical study leads to a defective interpretation of conditionals, that fact needs to be proved by means of further research. So far, we have no proofs that, certainly, that is the case. The discussion is open and other interpretations are also possible.

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